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On the Doppler radiation associated with Čerenkov radiation in the presence of an alternating electric field

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Abstract. The effect of an alternating electric field on the uniform superphase motion of a charge is to emit Doppler radiation along with the usual Čerenkov radiation. For relativistic velocities of the charge, general results, giving their intensities, are presented in compact form and analysed. It is found that the reduction in Čerenkov radiation is exactly balanced by the emission through Doppler radiation, i.e. the total energy radiated through the Čerenkov and Doppler radiations is always constant. Both the radiations show oscillatory behaviour.

1. Introduction

Following the suggestion made by Ginzburg (1947a, b) to use Čerenkov and Doppler effects to obtain microwave radiation, we have studied the radiation emitted when an alternating electric field $\boldsymbol{E} = \boldsymbol{E}_0 \sin \omega_0 t$ is applied first parallel and then perpendicular to the relativistic uniform linear superphase motion of a charge e through an infinite isotropic dielectric. The field superimposes oscillations on the linear motion of the charge. Consequently Doppler radiation (DR) (Frank 1943, Tavdgiridze and Tsintsadze 1970) is given out along with the usual Čerenkov radiation (CR). CR is mainly in the visible region, while discrete frequencies ω_θ emitted through DR are restricted to the lower-frequency region of the electromagnetic spectrum. Doppler frequencies depend upon the field frequency ω_0, the velocity of the incident charge, $\boldsymbol{v}_0 = \boldsymbol{\beta}_0 c$, and the dielectric. They are given by

$$\omega_\theta = \left| \frac{\pm l\omega_0}{\beta_0 n(\omega_\theta) \cos \theta - 1} \right|, \tag{1}$$

where $l = \pm 1, \pm 2 \ldots$, the refractive index $n(\omega_\theta) = (\epsilon(\omega_\theta)\mu(\omega_\theta))^{1/2}$, and θ is the angle of observation. DR is classified into 'normal' Doppler radiation (NDR) and 'anomalous' Doppler radiation (ADR). ADR is given out only at superphase velocity and is restricted to the region inside the Čerenkov cone. NDR is emitted at any velocity and is outside the Čerenkov cone.

The present paper is a sort of continuation of our previous paper (Risbud and Takwale 1977, referred to hereafter as I). Considering relativistic velocities of the charge, more general results for the intensities of CR and DR are given in simple closed forms and analysed. Dispersion is neglected. While calculating the intensity of DR, the field frequency is assumed to be very much less than the desired frequency, i.e. $\omega_0 \ll \omega_m$.

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2. Parallel field: results and analysis

Let $\boldsymbol{E} \| z$ and $\boldsymbol{v}_0 \| z$. We note that equation (4) of I applies to nonrelativistic motions of the charge. For relativistic motion it takes the form

$$v = \frac{v_0 - v_u \cos \omega_0 t}{1 - (v_0 v_u / c^2) \cos \omega_0 t} = v_z, \qquad v_x = v_y = 0 \tag{2}$$

where $v_u = eE_0 / m\omega_0$ and $m = m_0(1 - \beta_0^2)^{-1/2}$.

Following the treatment given in I, using equation (2), and making a linear approximation in v_u, we get the result for the intensity of the radiation as

$$I = I_C\left(1 + \frac{\lambda'}{2}\right) \sum_{l=-\infty}^{+\infty} \left(J_l^2(\lambda'\omega_m) - J_{l-1}(\lambda'\omega_m) J_{l+1}(\lambda'\omega_m) - \frac{2l\omega_0}{\pi\lambda'\omega_m^2} \frac{1}{(1 - 1/\epsilon\mu\beta\beta_0)} \right) \tag{3}$$

where

$$I_C = \frac{e^2\mu}{c^2} \frac{\omega_m^2}{2}\left(1 - \frac{1}{\epsilon\mu\beta\beta_0}\right), \qquad \lambda' = \frac{eE_0}{m\omega_0^2 v_0}(1 - \beta_0^2)^{3/2} = \lambda(1 - \beta_0^2)^{3/2}, \qquad \beta = \left|\frac{v}{c}\right|,$$

J_l, J_{l-1} and J_{l+1} are Bessel functions, ω_m is the maximum frequency in the electromagnetic spectrum up to which the CR condition is satisfied, and λ is as defined in I. For $l = 0$ equation (3) gives the intensity of CR (I_{CR}), and for $l \neq 0$ it gives the intensity of DR (I_{DR}). Positive integral values of l give the intensity of ADR, and negative integral values of l correspond to NDR. Equation (3) is similar to equation (19) of I, except for the fact that in equation (3) there is (i) an additional term of the order of λ', (ii) a multiplying factor, namely v_0^2/v^2, in I_C, and (iii) a multiplying factor, namely $(1 - \beta_0^2)^{3/2}$, in the argument of the Bessel functions. The first two factors have entered in the expression because here we have included terms of the order of v_u (which are neglected in I). Factor (iii) has come in as a correction for relativistic velocities. We note that the contribution from (i) and (ii) is negligibly small ($\lambda' \ll 1$, $v_0/v \approx 1$, since relativistic effects on the mass and velocity of the charge restrict the values of v_u and λ'). The correction given by (iii) is important because, in practical applications of the results, the refractive indices of most suitable radiator materials would demand the use of relativistic electron velocities.

From equation (3) we write

$$I_{CR} = I_C(1 + \lambda'/2)(J_0^2(x) + J_1^2(x)) \tag{4}$$

and

$$I_{DR} = I_{ADR} + I_{NDR} = 2I_C(1 + \lambda'/2) \sum_{l=1}^{\infty} (J_l^2(x) - J_{l-1}(x) J_{l+1}(x)), \tag{5}$$

where $x = \lambda'\omega_m$.

From equation (4) we see that in the presence of the parallel field: (i) CR has an oscillatory behaviour; (ii) the zeroes of the Bessel function $J_1(x)$ correspond to extremum intensity points; and (iii) there is an overall reduction in CR from its zero-field value.

From equation (5) we see that: (i) DR is a field effect; (ii) the intensity of the different modes of DR shows an oscillatory behaviour; and (iii) the zeroes of the Bessel function $J_{l-1}(x)$ give maximum intensity points, and the zeroes of $J_{l+1}(x)$ correspond to minimum intensity points of any Doppler mode.

As $x = \lambda'\omega_m = (eE_0/m_0\omega_0^2 v_0)(1-\beta_0^2)^{3/2}$ contains parameters E_0, ω_0 and v_0, the effect of each of them on the intensity of the radiations can be found out separately from equations (4) and (5).

Further, using the summation properties of Bessel functions (Gradshteyn and Ryzhik 1965) we write from equations (3), (4) and (5)

$$I_{\text{total}} = I_{\text{CR}} + I_{\text{DR}} = I_{\text{C}}(1+\lambda'/2). \tag{6}$$

Since $\lambda' \ll 1$ and $v \sim v_0$, equation (6) gives a very important result: that for superphase velocities the total energy radiated through CR and DR in the presence of a parallel alternating electric field is constant, and that it is nearly equal to the energy radiated through CR in the absence of the field.

In presence of the field, CR is reduced and DR is emitted. As the total energy radiated is constant, the reduction in CR is exactly balanced by the emission through DR. Since the frequency regions to which these radiations are restricted are distinctly separated, the ultimate effect of the parallel alternating electric field is to shift the radiation energy from the visible region to lower-frequency regions of the electromagnetic spectrum.

3. Perpendicular field: results and analysis

Let $\boldsymbol{E}\|\boldsymbol{x}$ and $\boldsymbol{v}\|\boldsymbol{z}$. For relativistic velocities we write the velocity of the charge in the presence of a perpendicular field as

$$v_x = \frac{-v_u(1-\beta_0^2)^{1/2}\cos\omega_0 t}{1-(v_0 v_u/c^2)\cos\omega_0 t}, \qquad v_y = 0, \qquad v_z = v_0. \tag{7}$$

Using equation (7), following the treatment given in I, and making a linear approximation in v_u, we get the same result for the energy loss of the charge per unit path length as given by equation (27) of I, except for a multiplying factor, namely $(1-\beta_0^2)$, in the argument of the Bessel function. Taking account of this change, we obtain from equations (27) and (28) of I the following results for the intensity by splitting the sum over l from $-\infty$ to $+\infty$ into two sums, namely $-\infty$ to 0 and 1 to ∞, substituting $\omega' = \omega \pm l\omega_0$, keeping terms up to the order of (ω_0/ω'), using the expansion of the hypergeometric function ${}_2F_3$, performing integration w.r.t. ω', and simplifying,

$$I_{\text{CR}} = \frac{I'_{\text{C}}}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\Gamma^2(n+\frac{1}{2})(\alpha x')^{2n}}{(n+1)\Gamma^4(n+1)} \tag{8}$$

$$I_{\text{DR}} = \frac{2I'_{\text{C}}}{\pi}\sum_{l=1}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n\Gamma^2(n+l+\frac{1}{2})(\alpha x')^{2n+2l}}{n!(n+l+1)\Gamma^2(n+l+1)\Gamma(n+2l+1)}, \tag{9}$$

where

$$I'_{\text{C}} = \frac{\mu e^2}{c^2}\frac{\omega_m^2}{2}\left(1-\frac{1}{\epsilon\mu\beta_0^2}\right), \qquad \alpha = (\epsilon\mu\beta_0^2-1)^{1/2},$$

$$x' = \frac{eE_0\omega_m}{\omega_0^2 v_0 m_0}(1-\beta_0^2) = \lambda\omega_m(1-\beta_0^2).$$

The infinite sums appearing on the right-hand sides of equations (8) and (9) are absolutely convergent. The intensities of ADR and NDR differ slightly. They are given by

$$I_{\mathrm{NDR}} = \tfrac{1}{2} I_{\mathrm{DR}}(1+d), \qquad I_{\mathrm{ADR}} = \tfrac{1}{2} I_{\mathrm{DR}}(1-d) \tag{10}$$

where $d = (2l\omega_0 \epsilon\mu\beta_0^2/\omega_m \alpha^2)(l+n+1)$. Evaluation of the coefficients of the infinite series appearing in equation (8) shows that it agrees exactly with equation (31) of I, except for a multiplying factor of $(1-\beta_0^2)^{2n}$ (which comes as a relativistic correction). We point out that the condition in equation (28) of I, namely $\mathrm{Re}(2l+1) > 0$, does not forbid NDR. Both ADR and NDR are given out in this case also. If the contribution from NDR is added to equation (33) of I, which takes into account ADR only, we see that it is a special case, namely $l = 1$, of the above result, equation (9), except for the fact that equation (9) contains an additional factor $(1-\beta_0^2)^{2n+2l}$. In the absence of the field, equation (8) reduces to the familiar result for CR obtained by Frank and Tamm (1937), and equation (9) gives no contribution to the radiation, showing that DR is a field effect. In equations (8) and (9) the terms $n = 0$ contribute to the radiations when the external field is not present, and the terms $n = 1, 2, \ldots$ account for the field effect.

Further, we have calculated for various values of x' the infinite sums appearing on the right-hand sides of equations (8) and (9) using a computer. The results are shown graphically in figure 1. From a numerical analysis of the above results we come to the conclusion that the perpendicular field also reduces CR and gives out DR in such a way that the total energy radiated through these two radiations is always constant, and this constant is equal to the energy radiated through CR in the absence of the field.

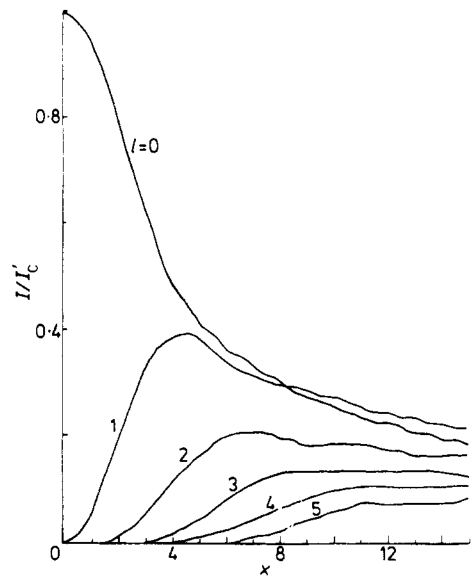

Figure 1. Variation of intensity of Čerenkov radiation ($l = 0$) and different modes of Doppler radiation ($l = 1$–5) for $\boldsymbol{E} \perp v_0$.

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